

CONTRAMODULES AND THEIR APPLICATION IN TILTING THEORY

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Abstract.

Contramodules are module-like algebraic structures endowed with infinite summation operations satisfying natural axioms. Introduced originally by Eilenberg and Moore in 1965 in the case of coalgebras over commutative rings, they didn't attract too much interest and they were forgotten for more than three decades.

About fifteen years ago, contramodules appeared again in the literature thanks mainly to Leonid Positselski's work. The motivation towards their study was in particular for their many applications in algebraic geometry. They were studied for the purposes of the semi-infinite cohomology theory and the comodule-contramodule correspondence.

In this series of lectures I will consider some instances of contramodules. The first basic examples will be objects of an abelian reflective category exactly embedded in a module category over a commutative ring. Another important example is provided by the abelian category of contramodules over a topological associative ring. This will lead to applications in tilting theory. In particular, when A is a module category over an associative ring (or a locally finitely presentable Grothendieck category) with an n -tilting object, the tilting heart B is equivalent to the abelian category of contramodules, over the topological ring of the endomorphisms of the tilting object and there is triangle equivalence at the level of the derived categories $\mathcal{D}(A)$ and $\mathcal{D}(B)$.

Everything is based on a series of papers by Leonid Positselski and coauthors available on the arXiv.