CONTRACTION ALGEBRAS: PROPERTIES, PROPHESIES AND PREDICTIONS

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This is a survey of recent work that aims to classify certain 3-dimensional geometric surgeries called 'flops' using noncommutative finite dimensional algebras. The hoped-for classifying objects are the contraction algebras of the title. There are various ways of defining them: via noncommutative deformation theory, idempotent factors of tilted algebras, certain cluster titled algebras, or by quivers with potentials. There is a caveat, in that the quivers always have two cycles, and almost always have loops: the baby case, which is still not fully understood, is the two-loop quiver.

Given these talks are taking place at ICRA, they will focus almost entirely on the purely algebraic aspects of this story: the definition of contraction algebras and universality, their known algebraic properties, and both the numerical and structural predictions that they make. The rough plan of the three lectures is as follows:

- Noncommutative Deformation Theory, and why you should care. Then this setup applied to structure sheaves of curves, and to the simples of an algebra. They give the same answer! The Contraction Theorem, which explains the significance of the contraction algebra being finite dimensional.
- The Classification Problem, and The Realisation Problem. Then some homological properties, including the role contraction algebras play in spherical twists and derived autoequivalence groups.
- Numerical Properties: towards an ADE classification of 'cluster theory' on the two-loop quiver. Slicing by a generic central element gives one of six algebras, and Toda's weighted sum formula, which gives an algebraic way of extracting the curve-counting Gopakumar–Vafa invariants.