Gradability of modules over graded artinian rings.

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Let Λ be a $\mathbb{Z}$-graded artin algebra. Two classical results of Gordon and Green [1] state that if Λ has only finitely many indecomposable gradable modules, up to isomorphism, then Λ has finite representation type; and if Λ has finite representation type then every Λ-module is gradable. We generalize these results to $\mathbb{Z}$-graded right artinian rings R. The key tool is a characterization of gradable modules: a f.g. right R-module is gradable if and only if its "pull-up" is pure-projective. Using this we show that if there is a bound on the graded-lengths of the f.g. indecomposable graded R-modules, then every f.g. R-module is gradable. As another consequence, we see that if a graded artin algebra has an ungradable module, then it has a Prüfer module which is not of finite type, and hence it has a generic module by work of Ringel [2].

References

[1] R. Gordon and E. L. Green. *Represenation theory of graded Artin algebras.* J. Alg. 76, (1982), 138–152.

[2] C. M. Ringel. *The relevance and ubiquity of Prüfer modules.* Representation theory, 163–175, Contemp. Math., 478, Amer. Math. Soc., Providence, RI, 2009.